技術文化論叢

第9号(2006年)

東京工業大学技術構造分析講座

『技術文化論叢』第9号(2006年)

目次

論文

Why Did Germany not Succeed in Achieving a Neutron Chain Reaction During World War II ? —From the Viewpoint of Nuclear Reactor Physics—
Yuzo FUKAI (深井佑造) 1

中国における日本の「植民地科学」(一)
—範囲・時代・史料・問題意識— 梁波 28

資料紹介

淺田常三郎先生と長岡半太郎先生とフリッツ・ハーバー先生
福井崇時 41

2005年度博士・修士論文梗概

<博士論文梗概>

明治期における技術者の分析
—近代技術確立をめぐる職人と職人的技術者— 恒川清爾 67

コンピュータにおけるGUI発展過程に関する研究 姜波 86

<修士論文梗概>

矛盾許容型論理PCL2の研究 斉田泰伸 116

The Study on Elementary Protothetic 董遠 120

1930年代のナチスドイツにおける数学の正当化
—数学教育の観点から— 宮本英司 124

論文

Why Did Germany not Succeed in Achieving a Neutron Chain Reaction During World War II ?

—From the Viewpoint of Nuclear Reactor Physics—

Yuzo FUKAI

Abstract

The German program to achieve a chain reaction during World War II started at almost the same time as the American program. Whereas the American CP-1 could initiate a chain reaction as early as December 1942, the German quest for nuclear reaction ended without a productive result. The last German experimental device failed in April 1945. This paper looks at reasons for this failure from the viewpoint of nuclear reactor physics. In those days, it was extremely difficult in any country to procure the least amount of materials necessary to produce a chain reaction because an industrial infrastructure to provide high quality materials was not yet established. The minimum amount of uranium or fissionable material was estimated by critical mass calculation which was dependent mainly on an optimal weight ratio of moderator to uranium. The American researchers developed an elementary nuclear reactor theory for that estimation, and succeeded in building CP-1 by using an optimized method of this theory. In spite the fact that M. F. Perrin's method, which is shown here to have an equivalent theoretical skeleton to the elementary reactor theory, had already been public before the German program started, the German researchers adhered to a different estimation method that they developed themselves, and then disregarded the optimization concept suggested in Perrin's paper. Furthermore they ignored the effect of reflector in their calculation of the critical mass, based upon the Haigerloch sub-critical experiment. Such reasons played a crucial role in their failure in nuclear chain reaction. This paper argues that their adherence to their own estimation method was a result of their superiority complex.

1 Introduction

During 1939 and 1940, most nuclear physicists understood that a fission chain reaction (by thermal neutron) in light water moderated natural uranium system was unfeasible. Then, for the sake of feasibility of the chain reaction in natural uranium system, they chose either graphite or heavy water that had a lower absorption thermal neutron cross section than light water as a moderator, and American physicists selected graphite, whereas German physicists, heavy water. In Germany, since the experimental results of W. Bothe clarified that the domestic graphite had more impurity, and a lot of heavy water could be obtained from the Norsk Hydro electrolytic company of Norway under the control of Germany, the Heereswaffennamt(the weapon development administration office) decided on heavy water as the moderator[1]. To the contrary, in the USA of the time, it was difficult to obtain a lot of heavy water, but a large amount of the high qualified graphite was easily obtainable.

In order to achieve a chain reaction as early as possible due to the rivalry for a nuclear development under the War conditions, it seems that the German plan was preferable to the American one, because heavy water has a lower absorption thermal neutron cross section than graphite. However, the fact of the matter is that the American graphite moderated pile: CP-1 has initiated the chain reaction in December 1942, and, even in April 1945, the chain reaction research failed in the last German experimental facility. Furthermore, the starting date of the both American and German programs was September 1939. Thus, in spite that it took about three years and half to achieve the chain reaction in the American case, the German program did not produce a breakthrough in its five and half year period, until the experimental facility was confiscated by the American army.

The reason that the German plan failed in the chain reaction is generally considered to be insufficient funding, or the total destruction of domestic manufacturing industries and transportation in a heavy air raid[2]. However, since twenty experiments carried out at many facilities to achieve a chain reaction until all the German research efforts were stopped[3], the author doubts if there was a nuclear physical reason except for the above-mentioned one for the German failure.

As generally explained in a previous paper[4], the underlying principle for a chain reaction physics is far simple. The balance of neutrons, namely, the rate of production is equal to a sum of the rate of absorption (disappearance) and leakage of neutron in a given volume of the neutron multiplying medium in a steady state. Then, the general equation representing this balance is shown as follows:

$$\text{(Production)} = \text{(Absorption)} + \text{(Leakage)} \tag{1}$$

In the Equation (1), Absorption contains the effect of neutron absorption to the fissionable material which occurs the neutron generation by the fission effect of this material in the medium. The amount of this neutron generation is (Production) term of the left side of Eq. (1). Thus, Eq. (1) implies a concept of neutron circulating cycle:

(neutron disappearance) $\longrightarrow$ (neutron generation) $\longrightarrow$ (neutron disappearance) $\longrightarrow$ (neutron generation) $\longrightarrow$ (neutron disappearance).....

Therefore, it is called 'fission (neutron) chain reaction' . Here is a definition of the effective multiplication factor: k_{eff},

$$k_{eff} = \frac{\text{(Production)}}{\text{(Absorption)} + \text{(Leakage)}} \tag{2}$$

since it is clear that $k_{eff} = 1$ is equivalent to Eq. (1), $k_{eff} = 1$ is shown as producing a chain reaction for a system to be discussed. In other words, the system becomes critical. Next, an important physical concept of the infinite multiplication factor, k_∞ can be defined as follows:

$$k_\infty = \frac{\text{(Production)}}{\text{(Absorption)}} > 1 \tag{3}$$

Since an actual system is always finite, the term (Leakage)>0. Then, Eq. (3) is derived from Eq. (2). Namely, $k_\infty > 1$: Eq. (3) is the necessary condition for achieving a chain reaction. In other words, if only a system meets Eq. (3) condition succeeds to be constructed, and

such a system is enlargened, the chain reaction should become feasible.

Since nobody can produce the Eq. (3) condition in the light water moderated natural uranium system, either graphite or heavy water was selected as the moderator. However, heavy water content is only about 0.015% in ordinary water, and also even marketable graphite generally shows high thermal neutron absorption effect. During World War II, it was in every countries extremely difficult to procure the necessary amount of the materials to achieve a chain reaction, because an industrial infrastructure to supply the materials of high quality was not yet established. Thus, all the nuclear researchers who intended to produce a chain reaction had to solve the issue of procuring the least necessary amounts of the moderator material and the natural uranium.

After World War II, Heisenberg reported several k_∞ values arranged from values of their experiments according to the definition which is described in the Smyth Report[5], as shown in his 'Anhang' report[3]. Then, the k_{eff} values of their experiments by using the arranged k_∞ values and their measured 'Vermehrungsfaktor' Z values which are the ratio of the total neutron leakage to the strength of neutron source can be estimated[6]. Table 1 shows the values of k_∞ and k_{eff} for the German experiments in order of year and comparison with the developments in USA.

Because of the success of Leipzig device meeting the Eq. (3) condition in September 1941, Germany seemed the first winner for the USA-Germany rivalry[7]. This German Leipzig device: L_4 had the uranium oxide and heavy water of 150 liter. However, a increasing rate of k_∞ value every year is 0.038/yr in Germany, since the value of k_∞ was 0.92 in 1940 and 1.11 in 1945, as shown in Table 1. On the contrary, the same increasing rate for the case of USA was 0.14/yr: about 4 times as fast as that of Germany, since k_∞ was 0.89 in August 1941 and 1.07 in December 1942 for the graphite moderated natural uranium system, and surely USA then won the USA-Germany rivalry by the success of its chain reaction. Furthermore, even in the case of the heavy water moderated natural uranium system, USA beat Germany in 1944, as shown in Table 1.

Table 1: German values of k_∞ and k_{eff} and comparison with that of USA

year	ex. name	k_∞	Z	k_{eff}	USA		remarks
1940	B_1	0.92	< 1	-			
1941	L_4	1.01	1.1	0.91			Leipzig device
1941					k_∞=0.89		Columbia pile
1942	G_2	1.09	1.37	0.82			
1942					k_∞=1.07	k_{eff}=1.0006	CP-1
1943	B_{6b}	1.08	2.35	0.95			
1944					k_∞=1.22	k_{eff}=1	F-Z reactor*
1944	B_7	1.08	3.06	0.97			
1945	B_8	1.11	6.7	0.98			Haigerloch device

In this Table, astarisk *, shows the world-wide first success of the chain reaction in a heavy water moderated natural uranium system by E. Fermi and W. Zinn, and 'ex. name' is the symbol of the German experiment.

This paper looks at, first, the Fermi-Zinn reactor which is described in the United States Patent # 2708656[8]. Then it will assess the R&D program that achieved $k_\infty > 1$ in Germany. Finally, it will discuss the reason for the German failure on the basis of the differences in the technology of nuclear physics in the USA and Germany during World War II.

2 R&D on Heavy Water Moderated System in USA

2.1 Fermi-Zinn Reactor for Construction of CP-3

In the patent application by Fermi and Zinn, the Fermi-Zinn reactor is explained as an experimental heavy water moderated natural uranium system for achieving a chain reaction. The constitution of the Fermi-Zinn reactor shown in 'Figure' of the vertical cross section described in the patent almost agree with the those of CP-3. It is from this fact that it can be considered that the Fermi-Zinn reactor was built as a model of CP-3 in order to prove a chain reaction in the heavy water moderated system. Since CP-3 was built in May 1944[9], it is sure that the date of the success of the chain reaction in this system came before that, and perhaps prior to the construction of Haigerloch device in Germany.

An approach to criticality for the Fermi-Zinn reactor was that a level of heavy water is raised to a height of 122.4 cm, being poured into the Al tank with the arrangement of 136 natural uranium rods. Then, the whole amount of the critical mass can be measured by such a method, only when a volume ratio of heavy water to uranium is beforehand determined. Although its method approaching to criticality is same as in the case of the CP-1, it is quite different from the case of the Haigerloch device. Namely, Fermi and Zinn already had a calculating method, by which a critical size for the system can be forecasted. This method is the elementary nuclear reactor theory[10] which was developed by Fermi's group during the construction of the CP-1. A schematized concept of the theory is quite similar to the one published[11] in May 1939 by M. F. Perrin as shown in Appendix A. It should be noted that the publication date of the Perrin's paper preceded the start of the rivalry.

The estimation of the above mentioned criticality of the Fermi-Zinn reactor by use of the elementary reactor theory is examined next[12]. First, k_∞ value: 1.22 of the Fermi-Zinn reactor is derived from 'Figure (with signatures of Fermi and Szilard) of the k_∞ values as parameters of the uranium rod radius and the volume ratio' described in the patent. Approximating to a sphere from the actual cylindrical shape of the Fermi-Zinn reactor, the sphericalized reactor has a radius of 92 cm with the thickness 71 cm of the graphite reflector. Using the elementary reactor theory[10], the critical size of the sphericalized reactor with the k_∞ value of 1.22 is calculated and the results are shown in Table 2, where the specifications of CP-3 is added for reference. It is then from the Table 2 clear that the calculated values by use of the elementary reactor theory agree well with the actual critical masses of uranium and heavy water of the Fermi-Zinn reactor.

Table 2: Specific of Heavy Water Moderated System of USA

item	critical radius	reflector thickness	graphite	uranium	heavy water	W/U*
F-Z reactor						
: actual	92 cm	71 cm	24.1 ton	1.85 ton	3.41 ton	1.84
: calculation	95 cm	70 cm	25.9 ton	2.07 ton	3.83 ton	1.84
CP-3	—	61 cm	—	3.0 ton	5.34 ton**	1.78

*: The weight ratio of heavy water to uranium.
**: The upper heavy water reflector has 1.16 ton=about 10 cm of the thickness, then the total amount is 6.5 ton[9].

2.2 Optimization

Since it was extremely difficult to procure the necessary amount of the materials to achieve a chain reaction in those days, as mentioned before, it was very significant to estimate a optimal volume ratio of moderator to uranium which decides mainly the nuclear properties of the reactor, especially the critical mass. E. P. Wigner and G. N. Plass attempted to calculate a 'optimal lattice' for the construction of CP-1 in early 1942[13]. They obtained a volume ratio, of which the core has the lattice with the highest value of k_∞. Calculation of the lattice with the highest value of k_∞ had already been proposed by L. Szilard in January 1940[14]. The optimization of the lattice was a common physical knowledge among Fermi's research group. Although they have not explained a physical reason that k_∞ has the highest value at a specific volume ratio of lattice, the physical property had been explained by O. Minakawa and H. Tamaki[15]. In Japan, such an optimization concept had been adopted in the estimation[16] of 'Ni Go study' operated by the Riken (the Institute of Physical and Chemical Research) during World War II. The Riken researchers developed this concept by studying Perrin's paper[11]. As shown in Appendix A, since his calculating method has the same theoretical skeleton as the elementary reactor theory, the Japanese optimization might be considered to have been essentially equivalent to that of USA at the time.

In the patent, there is the following description for the heavy water moderated natural uranium system, that is equal to the above-mentioned 'Figure'.

When heavy water is used as moderator, higher k_∞ approaching 1.3 is obtainable.

Since the range of the volume ratio which is satisfied with Eq. (3) condition in the heavy water moderated system is wider and the values of k_∞ is far over unity, it would be not necessarily adapted that the core has the highest value of k_∞ approaching 1.3. Then, k_∞ value of the Fermi-Zinn reactor of 1.22 was adopted but not the maximum value. However, all the k_∞ values of the German device shown in Table 1 seem to have been fairly lower than the ones given in the patent.

3 R&D for Heavy Water Moderated System in Germany

3.1 Organization in nuclear reactor physics in Germany

Research was carried out by the following 4 research groups,

research group	devices	researchers
(1) Physics Department of Leipzig University	$L_1 \sim L_4$,	K. Döpel, W. Heisenberg.
(2) Gottow Test Factory of Heereswaffennamt	$G_1 \sim G_3$,	F. Berhei, W. Czulius, K. Diebner, G. Hartwig, W. Herrmann.
(3) Central Laboratory of K.W.I. for Physics	$B_1 \sim B_8$,	F. Bopp, E. Fischer, W. Heisenberg, K. Wirtz.
(4) Heidelberg Branch of K.W.I.	no device,	W. Bothe, P. Jensen, O. Ritter.

A number of the total experiments was 20. $L_1 \sim L_4$ and $B_2 \sim B_5$ had a spherical shape. B_1 and $B_6 \sim B_8$ had a cylindrical shape with the same height as the diameter. Since an exact analytical solution can be derived only from a spherical configuration, it is better considered that the configuration to be estimated is a cylinder having the same height as the diameter. There is no detailed information for $G_1 \sim G_3$.

The German researchers understood that a system with $k_\infty > 1$ had possibility of a chain reaction, enlarging the size of this system, and then they might have thought about the possibility of achieving a chain reaction in a system, which had the same constitution as L_4, if heavy water of 5 ton and natural U of 10 ton were procured[17]. However, since such amounts were considered to be enormous and unavailable in those days, the onward R&D was afterward continued to be carried out for achieving a chain reaction in a system with the less amounts. Although the total 20 experiments then carried out, the results of the German R&D after achieving $k_\infty > 1$ are dealt with here.

3.2 Preliminary Experiments for Construction of Haigerloch Device

The experiments with a slab type fuel of $B_6 \sim B_7$ had been carried out at (3) in 1943-44 by Bothe and E. Fünfer. The results decided the weight ratio of heavy water to uranium (W/U) and also showed superiority of graphite over light water as a reflector. The results of the $B_6 \sim B_7$ experiments are shown in Table 3.

Table 3: Result of $B_6 \sim B_7$ experiment

ex. name		B_{6a}	B_{6b}	B_{6c}	B_{6d}	B_7
thickness of U	[cm]	1.0	1.0	1.0	2.0	1.0
uranium metal	[ton]	2.12	1.25	0.89	1.78	1.25
heavy water	[ton]	~1.5	~1.5	~1.5	~1.5	~1.5
W/U ratio		0.71	1.2	1.65	0.84	1.2
measured Z value		1.56	2.35	2.12	2.06	3.6

As explained later, the German researchers considered that a chain reaction can be expected to occur, when the measured Z value becomes infinite. The larger the Z value, the nearer approach to criticality. Perhaps with such a consideration, they decided the following issues for a new direction of R&D to achieve the chain reaction,

(i) A preferable value of the W/U ratio is about 1, because of the higher measured Z value.

(ii) Graphite reflector is more preferable.

(iii) Larger lump of the fuel is not necessarily preferable, seeing the case; B_{6d} of 2 cm thickness.

3.3 Outlines of Haigerloch Device: B_8

According to the above direction, each about 1.5 ton of natural uranium and heavy water are prepared for construction of a new device: B_8. Since it was proved, at Gottow Test Factory: (2) in the beginning of 1941 by the experiments of the heterogeneous lattice in the U-paraffin sphere, that a cube of the fuel is preferred to a slab type, the supplied uranium metal for Haigerloch device was mechanically processed to cubical shape. Although 6-7 cm of the side length may be preferable from theoretical consideration,

(iv) The side length of 5 cm is adopted, in the light of Gottow result and the above-mentioned (iii).

Then, the 680 cubical uranium fuels were supplied to Haigerloch site.

The whole size of Haigerloch device was the height and the diameter of 210 cm, including the graphite reflector of 10 ton with the thickness of 40 cm which perfectly surrounds a core. The core has the height and the diameter of 124 cm. The loaded cubical uranium fuels in the core are suspended in the regular intervals by 78 chains, of which each of the 40 chains has 9 fuels and one of the 38 chains has 8 fuels. Therefore, the actual total number of the loaded fuels is 664, and the amount is 1.49 ton. The heavy water was poured over the loaded fuels in the core, and then the total amount of the heavy water with the density of 1.1 g/cc was about 1.54 ton. It means that

Haigerloch core had the W/U ratios, weight of 1.03 and volume of 16.9. (4)

Furthermore, sphericalizing the actual Haigerloch device, the resultant configuration is as follows,

The radius of the core is 71 cm and the graphite reflector thickness is 50 cm. (5)

The procedure of constructing the Haigerloch device was remarkably different from the case of Fermi-Zinn reactor. If a criticality can not be clearly predicted, it should be necessary to have enough spare parts of the materials: fuel and moderator. Seeing such a procedure, perhaps one may think that Heisenberg expected to achieve a chain reaction in the Haigerloch device. However, the actual result of the Haigerloch device betrayed Heisenberg's expectation. The resultant Z is

$$Z = 6.7 \tag{6}$$

which was equal only to twice Z value of B_7. Thus, a goal of the way to achieve the chain reaction in Germany is considered to be far away.

3.4 A Summary of German Results after Achieving $k_\infty > 1$

In Table 4 are shown the specifications and experimental results of the device to achieve a chain reaction of heavy water moderated natural U system in the German project, similar to the one which was operated during and immediately after World War II in the countries except for Germany.

Table 4: Weight Ratio of Heavy Water to U (D_2O/U) for Each Device

	Device	U [ton]	D_2O [ton]	$(\frac{D_2O}{U})$	Z	References
Ger	L_2	0.142(U_3O_8)	0.164*	1.15	~ 1	(3) 1941
	L_4	0.755	0.164*	0.22	1.1	(3) 1942
	H.P.	3(plate)	1.5	0.5	?	(17)
	G_{2a}	0.189(cube)	0.189(ice)	1.0	1.65	(17)
	B_{6a}	2.12(plate)	1.5	0.71	1.56	(3) 1943
	B_{6b}	1.25(plate)	1.5	1.2	2.35	(3) 1943
	B_{6c}	0.89(plate)	1.5	1.65	2.12	(3) 1943
	B_{6d}	1.78(plate)	1.5	0.84	2.06	(3) 1943
	B_7	1.25(plate)	1.5	1.2	3.6(3.06)**	(3) 1944
	B_8	1.49(cube)	1.54	1.03	6.7	(3) 1945
	H.F.	2.30	2.37	1.03	$\infty : k_{eff} = 1$	(3)
USA	F-Z	1.85	3.41	1.84	$\infty : k_{eff} = 1$	(8) before 1944/5
	CP-3	3.0	5.34	1.78	$k_{eff} > 1$	(9) 1944/5
	NRX	10.5	17.0	1.62	$k_{eff} > 1$	(9) 1947/8
FRN	ZOE	3.0	4.6	1.53	$k_{eff} > 1$	(9) 1948/12
	P-2	3.3	6.3	1.61	$k_{eff} > 1$	(9) 1952/10
Nor	JEEP	2.42	7.0	2.89	$k_{eff} > 1$	(9) 1951

Note: * =150 liter; **: Z values of 3.6 in 'Table 3', and 3.06 in 'Table 4' of ref. (3).
Abbreviation: Ger=Germany, H.P. =Heisenberg's next plan, H.F. =Heisenberg's forecast, F-Z=Fermi-Zinn, FRN=France, Nor=Norway.

The following conclusion can be easily drawn from Table 4.

The weight ratio of heavy water to natural U (D_2O/U) for the German device was below 1.2, except for B_{6c}, and lower than all other countries.

Next, selecting the experimental results of the German device which meets the condition of Z>1 from Table 4, and, rearranging the Z and k_∞ value as function of D_2O/U, the following Table 5 is obtained,

Table 5: Rearranged Data from Table 4

D_2O/U	0.22	0.71	0.84	1.0	1.03	1.2	1.2	(1.65)
Z	1.1	1.56	2.06	1.65	6.7	2.35	3.6	(2.12)
k_∞	1.01	-	-	1.09	1.11	1.08	1.08	(-)
Device	L_4	B_{6a}	B_{6d}	G_{2a}	B_8	B_{6b}	B_7	(B_{6c})

Since the Z-value depends on variable specifications of the core and the reflector, its relation to the D_2O/U can not be so strictly discussed, but may be roughly considered to have a proportional increase. On the contrary, the k_∞ value can be considered to be in proportion to the D_2O/U, since it depends only on the specifications of the core, unless a form of the U is remarkable: the U form of G_{2a} and B_8 is a cube and the others are not so. It is clearly understood from Table 5 that the performance (k_∞ and Z values) of the devices to achieve a chain reaction was improved by increasing D_2O/U.

On the other hand, the construction date of the devices is put in chronological order, then,

$$L_4 \rightarrow G_{2a} \rightarrow B_{6a} \rightarrow B_{6b} \rightarrow B_{6c} \rightarrow B_{6d} \rightarrow B_7 \rightarrow B_8$$

the above order is quite irrelevant to the trend of increasing D_2O/U shown in Table 5. Therefore, the German researchers who engaged in the German project were perhaps not interested in the following physical assumption:

the core having moderately higher D_2O/U value is preferable for achieving a chain reaction.

The above fact means that the German project had not reached the stage of an optimization study for the weight ratio of heavy water to U, which was considered to be very important for a nuclear development of the USA Fermi's[13][14] and Japan RIKEN's research group[15][16].

In the German project, the stage of such an optimization study would not find, or this study could not be carried out. The reasons for this is that the German researchers had mainly used Heisenberg-Wirtz's method for the analysis and the physical interpretation of their experimental results, and the method was moreover unsuitable for the physical forecast. However, since they had not included the optimization study in their plan, they had selected a system with $D_2O/U = 1$ and only forecasted the chain reaction of the system having both D_2O and U of about 2 tons available in Germany of the time.

Seeing Table 4 and comparing the German experimental results with the actual operated ones in the countries except for Germany, the ratio of D_2O/U to be adopted for a future device in Germany might be speculative to be preferred to be over 1.5. Although French

ZOE has $D_2O/U = 1.53$, the each weight of the constituent materials was heavier than the critical mass because of a research purpose. Furthermore, considering the actual operating result of Fermi-Zinn reactor which had lower value than 2 tons as critical weight of U, the speculated specification of the future device to achieve a chain reaction in Germany would be perhaps as follows,

$$\text{U of about 2 [ton], heavy water of about 3 [ton], and } \frac{D_2O}{U} = 1.5. \tag{7}$$

Accordingly, it might be presumed that although a sufficient amount of U was secured (see B_{6a} in Table 4), about 1 ton of heavy water was short for achieving a chain reaction in Germany during World War II. It is also remarked[18] that the available total amount of heavy water in those days was 'never more than 2.5 ton'. Since the amount of heavy water sunk to a fjord by the Norwegians' attack was 613 liter (0.67 ton), if this attack had not taken place, an available total amount would have been over 3 tons. It would be then imaginable that the improvement of the weight ratio had changed the outcome of the German R&D.

As explained before, USA had succeeded the chain reaction of the heavy water moderated natural uranium system by use of the elementary reactor theory developed by themselves. It can be from Table 2 understood that the criticality can be predicted by this theory. Before the German R&D started in September 1939, Perrin had already published a paper[11], which calculated a method for the chain reaction of a homogeneous system in May of the same year. The theoretical skeleton of the Perrin's paper is equal to the elementary reactor theory, as showed in Appendix A.

However, the German researchers actually never used any foreign method, and adopted a domestic method[19] developed by Heisenberg, which is remarkably different from the elementary reactor theory or the Perrin's calculating method. This difference has been discussed by R. Persson[20] in the study on k_∞ measurement for light water moderated natural uranium systems. In his conclusion, the following opinion for the difference between Heisenberg-Wirtz's method and the elementary reactor theory which is equivalent to the Perrin's method is shown,

> The differential method (the elementary reactor theory or Perrin's method) is more straightforward than the integral method (Heisenberg-Wirtz's method). ..... In the integral method one has to apply several correction to the measured quantities, and it is very difficult to estimate the errors in these correction, which are found theoretically.

Nobody could have doubts as that Heisenberg-Wirtz's method is physically correct because it is fundamentally introduced from the neutron balance; Eq. (1) in a chain reacting system. And this fact could be clarified by Persson's results[20]. However, when a spatial behavior of neutrons in their basic concepts is mathematically described, 'differential' method is adopted in Fermi's or Perrin's case, and, on the contrary, Heisenberg-Wirtz's method has 'integral' type. Thus, the Persson's conclusion 'the differential method is more straightforward than the integral method' is derived. Since the effects of neutron absorption and fission within the system and neutron leakage from the system are all inseparably contained in the Heisenberg-Wirtz's equation as the measured values, 'differential' method is generally thought to be superior to 'integral' type for discussion on a fair prospect of success to achieve a chain

reaction. Namely, Heisenberg-Wirtz's method would be considered to have been unsuitable for an optimization study.

By using nuclear data which are considered to reappear the German technology level in those days as faithfully as possible[12] and the elementary reactor theory, a result which can be justified to be nearly able to reappear the main properties of the German device is shown in Table 6.

Table 6: Comparison in Main Properties of Haigerloch Device

item	measure	calculation	calculation/measure
infinite multiplication factor: k_∞	1.11	1.14	1.03
'Diffusionslänge' [cm]	36.3	38.0	1.05

3.5 Heisenberg's Forecast from the Haigerloch Results

Heisenberg had insisted that the experiment results of the Haigerloch device indicated a chain reaction in Germany was possible. Since the details of his developed estimation is explained in Appendix B, his forecast and its shortcoming are briefly described here.

First, Heisenberg gave the following equation as a peculiar boundary condition between the core and the reflector at the radius: R of the sphericalized core,

$$\frac{d}{dr}(nr) = -B\gamma(nr) \qquad (\gamma = const) \tag{8}$$

where, r is a radial coordinate of the sphericalized core, n is a neutron density of the core, and B is the buckling: the reciprocal of 'Diffusionslänge'. By using the above boundary condition, a value of Z can be derived from Eq.(B9) of n in Appendix B and the definition of Z as follows,

$$Z = \frac{1 + \gamma BR}{cos(BR) + \gamma sin(BR)} \tag{9}$$

Since the critical condition: achieving the chain reaction is 'Z $\rightarrow \infty$', Heisenberg had derived the following critical equation from Eq. (9),

$$cot(BR) = -\gamma \tag{10}$$

Then, Heisenberg had forecasted a critical mass of a next chain reacting device to be constructed in Germany from the Haigerloch results: Z=6.7 and 'Diffusionslänge' by using Eqs. (9) and (10). Although Heisenberg had said that 'Diffusionslänge' is about 35 cm, the value of 'Diffusionslänge': 1/B is derived from Eq. (9) and γ=0.824 as follows,

$$\frac{1}{B} = 36.3 \ \ [cm] \tag{11}$$

Therefore, since BR=2.26 is obtained by Eq. (10) with γ=0.824, a forecasted critical radius becomes 82 cm from Eq. (11), and is 1.15 times the radius: 71 cm of the Haigerloch device. Converting these values to the critical mass of the next future chain reacting device to be constructed,

natural uranium; 2.30 [ton], heavy water; 2.37 [ton] (12)

are obtained. Thus, Heisenberg had insisted that they would succeed the chain reaction, if the both materials of about 2 tons were available. As shown in Table 3, Heisenberg surely had 2.12 ton of uranium for the experiment of B_{6a}. With regard to the available amount of heavy water in Germany, the following words can be quoted from Heisenberg's paper[2],

> In October 1943 the plant (Norsk Hydro electrolytic) was completely destroyed in a heavy air raid. Nevertheless, about two tons of heavy water were available in Germany at the time: a quantity which, according to our calculations (Eq. (12)), was just enough for the construction of an energy-producing pile. ..... A branch of the Reichsforschungsrat at Stadtilm was allotted the remaining quantity of heavy water and a great part of the available uranium. ..... The materials available at Haigerloch, however, was single insufficient to attain Z=∞. A relatively small amount of uranium would in all probability have sufficed; but it was no longer possible to obtain it, since transport from Berlin or Stadtilm could no longer reach Hechingen.

Heisenberg's insistence is considered as follows: the German researchers did not admit that their chain reaction ended in failure, because they could procure the least necessary amount of the materials to achieve a chain reaction. However, they could not in fact produce a chain reaction, owing to disturbance of their research by a heavy air raid which destroyed completely the domestic transportation network.

The Heisenberg's forecast above is however physically doubtful. Especially, the assumption of (γ=constant) in Eq. (8) would be correct in physics ?. Since it is physically correct that neutron density and current of a core and the ones of a reflector should have continuity at the boundary: r=R between the both sides, the following equation for γ is exactly derived from the above boundary condition to be physically appreciated, as in detail explained in Appendix B,

$$\gamma = \frac{1}{BR}\left(\frac{D_r}{D_c}\left(1+\kappa_r R\, coth(\kappa_r T)\right)-1\right) \tag{13}$$

where, T is the thickness of a reflector, D_c and D_r are diffusion coefficient of a core and a reflector respectively, and κ_r is the reciprocal of the reflector diffusion length. Therefore γ is clearly dependent on the specification and the nuclear properties of the given core and reflector, and can be not applied freely to another case as the constant value. Furthermore, combining Eq. (13) and Eq. (10),

$$BR\, cot(BR) = 1 - \frac{D_r}{D_c}\left(1+\kappa_r R\, coth(\kappa_r T)\right) \tag{14}$$

is obtained. The above equation is a critical equation of a spherical reactor with reflector given by M. F. Perrin[21], and also described in the elementary reactor theory[10]. The German researchers of those days should have sufficiently considered that the thickness of a reflector remarkably affects criticality of a chain reacting system, since Perrin already pointed out that the fast neutron critical mass of 40 ton without reflector could be reduced to 12 ton by using a pure neutron scattered material of Fe with the thickness of 35 cm as the reflector[21].

Eq. (8) given by Heisenberg is very 'grobe' approximate as said by himself. According to Heisenberg's report of December 6 in 1939[22], he considered that a reflector (an outside of

core) thickness is infinite and diffusion coefficients of the core and reflector are always equal. Applying this approximations: $T \rightarrow \infty$ and $D_r = D_c$ to Eq.(13), the following equation is given for γ,

$$\gamma = \frac{\kappa_r}{B} = constant \tag{15}$$

Since the actual Haigerloch device had a finite thickness of the reflector and different nuclear properties in the both regions, Eq.(13) should be adopted in the forecasting calculation from this experimental results, instead of (γ=constant). Since the detail of the more correct forecasting calculation is shown in Appendix B, only this brief reference is expressed here. Then, substituting Eq. (13) to Eq. (9), the following equation is given as

$$cos(BR) + \frac{\Delta - 1}{BR} sin(BR) = \frac{\Delta}{Z} \tag{16}$$

where, Δ is

$$\Delta = \frac{D_r}{D_c} (1 + \kappa_r R coth(\kappa_r T)) \tag{17}$$

Although the neutron-physical information for graphite which had been used in the Haigerloch reflector is very little known, since the material is a single composition and does not consist of any component, this physical specification is considered to be easier estimated than the diffusion coefficient: D_c of the core consisting of two components: natural U and heavy water. Then, only D_c value can be here estimated from the Haigerloch experimental results by using Eqs.(16) and (17). Giving here the 3 cases of nuclear data set: D_r and κ_r as the graphite which might be used for Haigerloch reflector and using Haigerloch experimental results: 'Diffusionslänge' (1/B)=36.3 cm, Z=6.7, R=71 cm and T=50 cm, D_c values are derived from the above-mentioned procedure. Finally, from the critical equation: Eq. (14) with (1/B)=36.3 cm, D_c, D_r and κ_r as the input data, a critical radius: R_k can be calculated as the function of a graphite reflector thickness: T. Thus, some results are given in Table 7.

Table 7: Forecasted Criticality from Haigerloch Experimental Results

R_k [cm]	T [cm]	natural U [ton]	heavy water [ton]	graphite [ton]
84.5	42	2.55	2.62	10.1
82.5	50	2.37	2.44	12.6

It can be from comparison with Eq. (12) understood that the Heisenberg's forecasting method (γ=constant) from the Haigerloch result is optimistic for criticality estimation, and disregarded an important effect of a reflector to the critical mass. This optimistic result is owing to the Heisenberg's concept for the reflector effect: 'a reflector (an outside of core) thickness is always infinite and diffusion coefficients of the core and reflector are always equal'.

4 Why did Germany not Succeed in Chain Reaction ?

Many believe that German researchers failed to produce a chain reaction because they could not estimate correctly minimum least necessary amount of materials to do so. Why did they not adopt the Perrin's calculating method that had already been made public in France, which was equivalent to the American elementary reactor theory ? Why did they adhere to Heisenberg-Wirtz's method developed by themselves ? Because of their peculiar method, they could not derive the concept of optimization and procure the least necessary materials. Perhaps, this is why German failed. In Japan, the Perrin's calculating method was used in the estimation of 'Ni Go study' operated by the Riken, and the concept of optimization was also adopted. S. Weart has published[23] a survey of on the world-wide history of R&D for chain reaction, and pointed out that F. G. Houtermanns had in August 1941 almost derived a complete equation of the four-factor formula which is equal to Eq. (A24) of Appendix A. He suggested Houtermanns' result did not sufficiently contributed to the German R&D, because he did not join the Heisenberg's research group. Furthermore, with regard to the forecasting from the Haigerloch result, they disregarded Perrin's finding about the effect of reflector[21], and adopted the 'grobe' approximation of (γ=constant). For such facts that the German researchers had repeatedly disregarded the Perrin's method, Weart has also described as follows,

> In the mid-1940 the capture of Paris allowed the Germans to analyze the French experimental procedures, although they never learned the theoretical tools the French had developed to analyze them.

and furthermore quoted the following words of two members of the German researchers: E. Bagge and K. Diebner after the War,

> While this theory (Heisenberg-Wirtz's method) is to a certain extent outstripped by another type of formulation (American elementary reactor theory) made by American physicists, it nevertheless remains correct in its essential points, and even today it would in principle suffice for calculation of a uranium reactor.

From the problems to be discussed here, Weart's impression mentioned above, and the over-confidence of two members of the German team, one may suggest that they had a strong sense of superiority to other nations and consequently believed they alone can conduct their experiments. Therefore, the author thinks that one strong reason for the German failure in chain reaction related to this strong superiority complex of German scientists.

Appendix A:

The Relation between Perrin's Equation and Elementary Reactor Theory

M. F. Perrin described that a possibility for a neutron chain reaction by the uranium fission had been soon discussed[21] after the fission had been discovered. First, Perrin proposed a critical calculating method of so-called 'one group diffusion theory' which can be applied mainly to a fast neutron chain reaction, namely, neutron energy generated by the fission is assumed to be the same as the thermal energy that the fission occurs. At the same time, many researchers; S. Flügge[24], M. F. Adler[25], and R. Peierls[26] described a physical concept of a neutron chain reaction and proposed a critical calculating method under the same assumption 'one group diffusion theory'. In May 1939, M. M. H. Halban, L. Kowarski, P. Savitch[27] first obtained a experimental value of ^{238}U resonance neutron absorption during a slowing down to thermal energy that the fission occurs from generated fast neutron energy by the fission. By their experimental result; p of the ^{238}U resonance escape probability, the theorist Perrin could derive a well understandable critical equation of so-called 'two group diffusion theory'[11]. Here, the theoretical skeleton of Perrin's proposed critical equation for homogeneous chain reacting system is presented to be equivalent to the one described in the elementary reactor theory[10].

Perrin shows a set of the equations in critical condition for fast neutron density: n_{ft} and thermal one: n_{th}. Since averaged neutron velocities for fast and thermal neutrons are v_{ft} and v_{th} respectively, each neutron fluxes are expressed by Φ_{ft} and Φ_{th} as follows,

$$\Phi_{ft} = n_{ft}\, v_{ft} \qquad \Phi_{th} = n_{th}\, v_{th} \tag{A1}$$

Accordingly, the original Perrin's critical equations can be rearranged by Equation (A1) to the following neutron balanced critical equations for two energy groups,

$$D_{ft} \bigtriangledown \Phi_{ft} + C_1 \Phi_{ft} + C_2 \Phi_{th} = 0 \tag{A2}$$

$$D_{th} \bigtriangledown \Phi_{th} + C_3 \Phi_{ft} - C_4 \Phi_{th} = 0 \tag{A3}$$

where, diffusion coefficients: D_{ft} and D_{th} are shown by each averaged mean free paths: λ_{ft} and λ_{th} respectively as follows,

$$D_{ft} = \frac{\lambda_{ft}}{3} \qquad D_{th} = \frac{\lambda_{th}}{3} \tag{A4}$$

and, since Perrin also considers the system that is a homogeneously mixed up medium of moderator and uranium, the coefficients: C_1, C_2, C_3, C_4 in Eqs. (A2) and (A3) are derived from Eq. (A1) and Eq. (A4) as follows,

$$C_1 = \left((\nu^{ft} - 1)\, N_U\, \sigma_{aU}^{ft} - sum_i\, N_i\, \sigma_{ai}^{ft} \right) - N_H\, \sigma_{SD} \tag{A5}$$

$$C_2 = \nu^{th}\, N_U\, \sigma_{aU}^{th} \tag{A6}$$

$$C_3 = p\, N_H\, \sigma_{SD} \tag{A7}$$

$$C_4 = N_U\, \sigma_{aU}^{th} + N_H\, \sigma_{aH}^{th} + sum_i\, N_i\, \sigma_{ai}^{th} \tag{A8}$$

where, ν: an average emitted neutron number by a fission of uranium, N: a nucleus number per a unit volume, σ: a neutron reaction cross section (unit: barn), and 'ft' and 'th' of an upper suffix show fast and thermal neutron energy respectively. With regard to a lower suffix of N and σ, U: uranium, H: moderator, i: nuclei except for U and H, a: neutron absorption, SD: slowing down. '$\mathrm{sum_i N_i \sigma_i}$' shows the summation of neutron reaction rate of nuclei except for U and H, and p is the resonance escape probability of ^{238}U which is obtained by an experiment.

From Eq. (A2) and Eq. (A3), an expression only for Φ_{ft} is derived by canceling Φ_{th}, and, by the same mathematical procedure, the one only for Φ_{th} is derived. Then, the two equations have quite same form with the same 4 coefficients: C_1, C_2, C_3, C_4. Accordingly, since the spatial distribution (not absolute value) for Φ_{ft} and Φ_{th} is quite same, an eigenvalue for either Φ_{ft} or Φ_{th} becomes same when Eq. (A2) and Eq. (A3) are solved, i.e.,

$$\nabla \Phi_{ft} + B^2 \Phi_{ft} = 0 \qquad \nabla\, \Phi_{th} + B^2 \Phi_{th} = 0 \tag{A9}$$

where, B is the eigenvalue, and is called as the 'buckling' in the elementary reactor theory.

Then, a total neutron absorption rate: Σ_{a} in the whole system can be expressed by

$$\Sigma_a^{ft} = sum_j\, N_j\, \sigma_{aj}^{ft} \qquad \Sigma_a^{th} = sum_j\, N_j\, \sigma_{aj}^{th} \tag{A10}$$

where, the summation of neutron reaction rate: '$\mathrm{sum_j}$' with the suffix 'j' in the above equation is carried out for all nuclei including U and H. Therefore, by use of Eq. (A10), C_1 and C_4 can be changed to the following expressions,

$$C_1 = -\left(N_H \sigma_{SD} - \left(\nu^{ft} N_U \sigma_{aU}^{ft} - \Sigma_a^{ft}\right)\right) \tag{A11}$$

$$C_4 = \Sigma_a^{th} \tag{A12}$$

Next, the fast fission effect: ϵ is derived as follows. Since a number of fast neutron generation by fast neutron fission is $\nu^{\mathrm{ft}} \mathrm{N_U} \sigma_{\mathrm{aU}}^{\mathrm{ft}}$, the net fast neutron number in unit volume of the system is reduced to $(\nu^{\mathrm{ft}} \mathrm{N_U} \sigma_{\mathrm{aU}}^{\mathrm{ft}} - \Sigma \mathrm{a}^{\mathrm{ft}})$ due to fast neutron absorption effects of all nuclei including U and H. The net fast neutrons suffer slowing down by moderator: $\mathrm{N_H} \sigma_{\mathrm{SD}}$, and they lost high energy and the fast fission effect. Therefore, a rate of increased fast neutrons before the suffering slowing down by moderator: Δ is expressed as follows,

$$\Delta = \frac{\left(\nu^{ft}\, N_U\, \sigma_{aU}^{ft} - \Sigma_a^{ft}\right)}{N_H\, \sigma_{SD}} \tag{A13}$$

It is noted that $\Delta << 1$. Then a fast neutron cycle occurs in the fast neutron energy group by a factor of Δ, and the increasing rate: ϵ of fast neutrons in the energy group is considered to be

$$\epsilon = 1 + \Delta + \Delta^2 + \Delta^3 + \Delta^4 + \cdots \tag{A14}$$

This effect adds to the whole neutron cycle in the chain reacting system. Since the fast neutron velocity is 8×10^9 cm/s (an averaged fission neutron energy: 2 MeV), a typical fast neutron cycle time is about 10^{-8} sec. While the thermal neutron velocity is 2.2×10^4 cm/s, and a typical neutron cycle time of thermal reactors is about 10^{-4} sec, then the fast neutron

cycle occurs about 10^4 times the whole neutron cycle. Therefore, Eq. (A14) is considered to be an infinite series, i.e.,

$$\epsilon = \frac{1}{1-\Delta} \tag{A15}$$

Then, Eq. (A11) becomes as follows,

$$\begin{aligned} C_1 &= -N_H\,\sigma_{SD}\left(1 - \frac{\nu^{ft}\,N_U\,\sigma_{aU}^{ft} - \Sigma_a^{ft}}{N_H\,\sigma_{SD}}\right) \\ &= -N_H\,\sigma_{SD}(1-\Delta) \\ &= \frac{-N_H\,\sigma_{SD}}{\epsilon} \end{aligned} \tag{A16}$$

It should be noted that ϵ of Eq.(A15) may be slightly different from a definition of the elementary reactor theory.

Since the coefficients: C_1, C_2, C_3, C_4 in Eqs. (A2) and (A3) are then Eqs. (A16), (A6), (A7) and (A12) respectively, renewed neutron balanced critical equations are derived as follows,

$$D_{ft} \bigtriangledown \Phi_{ft} - \left(\frac{N_H\,\sigma_{SD}}{\epsilon}\right)\Phi_{ft} + \nu^{th}\,N_U\,\sigma_{aU}^{th}\Phi_{th} = 0 \tag{A17}$$

$$D_{th} \bigtriangledown \Phi_{th} + p\,N_H\,\sigma_{SD}\Phi_{ft} - \Sigma_a^{th}\Phi_{th} = 0 \tag{A18}$$

Dividing Eq. (A17) by ($N_H\sigma_{SD}/\epsilon$) and Eq. (A18) by Σ_a^{th} and introducing the following constants by the definition of the elementary reactor theory,

$$\tau = \frac{D_{ft}}{N_H\,\sigma_{SD}} \tag{A19}$$

$$L^2 = \frac{D_{th}}{\Sigma_a^{th}} \tag{A20}$$

Eq. (A17) and (A18) are written as follows,

$$\epsilon\tau \bigtriangledown \Phi_{ft} - \Phi_{ft} = -\left(\frac{\nu^{th}\,N_U\,\sigma_{aU}^{th}}{N_H\,\sigma_{SD}}\right)\Phi_{th} \tag{A21}$$

$$L^2 \bigtriangledown \Phi_{th} - \Phi_{th} = -\left(\frac{p\,N_H\,\sigma_{SD}}{\Sigma_a^{th}}\right)\Phi_{ft} \tag{A22}$$

Multiplying Eq. (A21) by Eq. (A22), and applying its product to the following equation of η f and k_∞ in the elementary reactor theory,

$$\eta\,f = \frac{\nu^{th}\,N_U\,\sigma_{aU}^{th}}{\Sigma_a^{th}} \tag{A23}$$

$$k_\infty = \epsilon\,p\,\eta\,f \tag{A24}$$

the following resultant equation is obtained,

$$\epsilon\tau L^2 \bigtriangledown \Phi_{ft} \bigtriangledown \Phi_{th} - \epsilon\tau\Phi_{th} \bigtriangledown \Phi_{ft} - L^2\Phi_{ft} \bigtriangledown \Phi_{th} + \Phi_{ft}\Phi_{th} = k_\infty\Phi_{ft}\Phi_{th}$$

Dividing the both side of the above equation by $\Phi_{\rm ft}\Phi_{\rm th}$, since $\Phi_{\rm ft}\Phi_{\rm th} \neq 0$

$$\epsilon\tau L^2 \frac{\nabla\Phi_{ft}}{\Phi_{ft}} \frac{\nabla\Phi_{th}}{\Phi_{th}} - \epsilon\tau \frac{\nabla\Phi_{ft}}{\Phi_{ft}} - L^2 \frac{\nabla\Phi_{th}}{\Phi_{th}} + 1 = k_\infty$$

is obtained. Since the following relations are then derived from Eq. (A9),

$$\frac{\nabla\Phi_{ft}}{\Phi_{ft}} = \frac{\nabla\Phi_{th}}{\Phi_{th}} = -B^2 \tag{A25}$$

using Eq. (A25) to the above equation, one gets an equation:

$$\epsilon\tau L^2 B^4 + (\epsilon\tau + L^2)B^2 = k_\infty - 1$$

Here, with regard to fast neutron leakage from the chain reacting system, $\epsilon\tau \simeq \tau$ is used to be approximated. Since a total effect of leakage form the chain reacting system does not only contain the one of fast neutron but also the one of thermal neutron, the approximation is quite reasonable for estimation of the criticality. Accordingly, the above equation becomes,

$$\tau L^2 B^4 + (\tau + L^2)B^2 = k_\infty - 1 \tag{A26}$$

Since the migration area: M^2 in the elementary reactor theory is given as

$$M^2 = L^2 + \tau \tag{A27}$$

Eq. (A26) is rearranged to

$$\tau L^2 B^4 + M^2 B^2 = k_\infty - 1 \tag{A28}$$

For a large chain reacting system, the first term of the above equation can be neglected, since B^4 becomes extremely small. Therefore, Eq. (A28) becomes a simple form of $M^2B^2 = k_\infty - 1$. Then, considering Eq. (A25) again, the thermal neutron flux: $\Phi_{\rm th}$ for the large system can be calculated by the following equation,

$$\nabla\Phi_{th} + B^2\Phi_{th} = 0 \tag{A29}$$

where,

$$B^2 = \frac{k_\infty - 1}{M^2} \tag{A30}$$

Heisenberg had used Eq. (A29) as the critical equation: Eq. (B1) in Appendix B. However, he had not known that this equation can be derived from Perrin's paper.

The approximation for the neglected first term of Eq. (A28) may be reasonable, if the radius of spherical chain reacting system is far over the following value, 2.00~4.70 cm for the case of a light water moderated system, 34.4~35.7 cm for a heavy water system, and 34.2~51.8 cm for a graphite system.

Since only the light water moderated chain reacting system was considered when Perrin's paper had been published[11], the neutron balanced critical equation; Eq. (A2) and (A3) for two energy groups had been solved by an approximation of $D_{\rm th} \nabla \Phi_{\rm th} = 0$. Although this approximation is of course physically reasonable for case of the light water moderated system, such the method must not be applicable for any kind of moderator. However, as proved here,

the applicable calculating method can be derived from the original Perrin's paper, which is considered far exactly to include almost conceptions of physical phenomena for a chain reaction. The theoretical skeleton of the elementary reactor theory developed by Fermi's group can be then considered to have been created by Perrin, but his paper is not quoted in 'other references' of the Fermi-Szilard's patent, in spite that S. Flügge's paper has been quoted[8]. Fermi might have not understood what Perrin's paper had anticipated.

Appendix B:

Heisenberg's Forecasting Method for Critical Mass

German researchers always used the calculating method[19] which was developed by Heisenberg's group for estimation of their research results. In this appendix, an improved method for forecasting of a critical mass from the Haigerloch subcritical experiment is offered, since Heisenberg's forecasting method has the 'grobe' approximation described by himself, which is shown in Eq. (8) of the text.

A spherical system having a neutron source: Q at the center of a core with the radius: R and a reflector with the thickness of T is generally considered under the following conditions,

(I) Since the Haigerloch device was actually cylindrical with same dimension of the diameter and the height, the experiment should be considered to be sphericalized. Then, the sphericalized core has a radius of 71 cm, which can be perfectly surrounded with the graphite reflector of the thickness of 50 cm due to the total weight of 10 ton.

(II) A neutron behavior in the core is calculated by the diffusion theory. In Heisenberg-Wirtz's report[3], the following 'Diffusionsgleichung (7)' is expressed as a basic equation,

$$\nabla n_c + B^2 n_c = 0 \tag{B1}$$

where, n_c is a neutron density per a unit volume of the core, B is the buckling, a reciprocal of which is called 'Diffusionslänge' by Heisenberg. For the sphericalized system, Eq. (B1) is expressed in the following form as r is radial coordinates,

$$\frac{d^2}{dr^2}(n_c r) + B^2(n_c r) = 0 \tag{B2}$$

(III) A neutron behavior in the reflector is also calculated by the diffusion theory. A neutron density: n_r in the reflector is derived from the boundary condition of $n_r = 0$ at $r = R + T$ as follows,

$$n_r r = E\, sinh\,(k_r(R + T - r)) \tag{B3}$$

where, κ_r is a reciprocal of the diffusion length of the reflector and E is an arbitrary constant.

A general solution of Eq. (B2) with arbitrary constants of C and α is derived as follows,

$$n_c r = C\ (\alpha\, sin(Br) + cos(Br)) \tag{B4}$$

A physically reasonable boundary condition is the continuation of the neutron density and current at the boundary between the core and the reflector: r=R, i.e.,

$$n_c = n_r \qquad D_c\left(\frac{dn_c}{dr}\right) = D_r\left(\frac{dn_r}{dr}\right) \tag{B5}$$

where, D_c and D_r is the diffusion coefficient of the core and the reflector respectively. Substituting Eqs. (B3) and (B4) into Eq. (B5) and algebraically arranging this equation, the constant α is derived as,

$$\alpha = \frac{sin(BR) - \gamma\, cos(BR)}{cos(BR) + \gamma\, sin(BR)} \tag{B6}$$

where, an equation of γ is given as follows,

$$\gamma = \frac{1}{BR}\left(\frac{D_r}{D_c}\left(1 + \kappa_r\, R\, coth(\kappa_r T)\right) - 1\right) \tag{B7}$$

Since, instead of Eq. (B5), the quite same form of α-equation as Eq. (B6) is derived from substituting Eq. (B4) into Eq. (8) of the text, γ of Eq. (B7) is quite equivalent to the physically approximate concept of Heisenberg's forecasting method. Namely, γ of Eq. (B7) can be considered to improve the 'grobe' approximation of (γ=constant).

Since a neutron source with strength Q is located at the center of the core, the arbitrary constant C in Eq. (B4) is derived as follows,

$$Q = 4\pi|\mathrm{Lim}\ \mathrm{r}^2\mathrm{J_c}|_{\mathrm{r}\to 0} \qquad \mathrm{J_c} = -\mathrm{D_c}\frac{\mathrm{dn_c}}{\mathrm{dr}} \tag{B8}$$

After some algebraic procedures, $A = Q/4\pi D_c$ is given. Thus, the final equation of $n_c r$ is

$$r\, n_c = \frac{Q\ (\alpha\, sin(Br) + cos(Br))}{4\pi D_c} \tag{B9}$$

It should be noted that 'n' in the denominator of the right side of the equation 'n(r)' described in the Heisenberg-Wirtz's report is a misprint of 'r'. Next, since a neutron leakage: $< J >$ from the core to the reflector is similarly derived as $< J >= 4\pi|\mathrm{Lim}\ \mathrm{r}^2\mathrm{J_c}|_{\mathrm{r}\to\mathrm{R}}$, using a definition of $Z =< J > /Q$ and Eq. (B9), the measuring 'Vermehrungsfaktor' Z value described in the report becomes

$$Z = \frac{1 + \gamma BR}{cos(BR) + \gamma\, sin(BR)} \tag{B10}$$

Heisenberg had proposed the following critical equation from Eq. (B10) at $Z \to \infty$,

$$cot(BR) = -\gamma \tag{B11}$$

Furthermore, combining Eq. (B7) with Eq. (B11), a critical equation for a sphere reacting system with reflector is derived as follows,

$$BR\, cot(BR) + \frac{D_r}{D_c}\left(1 + \kappa_r\, R\, coth(\kappa_r\, T)\right) = 1 \tag{B12}$$

The above equation was published in Perrin's paper[21] or the elementary reactor theory[10].

The above-mentioned equations: Eq. (B6) of α, Eq. (B9) of rn_c and Eq. (B10) of Z have

the quite same form as described in Heisenberg-Wirtz's report[3], except for Eq. (B7) of γ.

Another paper by Heisenberg[22] described in December 6, 1939 shows that a neutron density: n_r in the reflector is as follows,

$$n_r r = E\, exp(-\kappa_r r) \tag{B13}$$

(a) the above equation is clearly derived from an approximation of (R+T)$\rightarrow \infty$ in the condition equation (III): Eq. (B3). By using the above equation, a value of γ is similarly derived as follows,

$$\gamma = \frac{1}{BR}\left(\frac{D_r}{D_c}(1+\kappa_r R) - 1\right) \tag{B14}$$

It is clear that Eq. (B14) is the case: $\kappa_r T \rightarrow \infty$ of Eq. (B7).

(b) If $\kappa_r r << 1$ in the whole range of r, the following equation is derived from Eq. (B13),

$$n_r r = E \tag{B15}$$

which is the Heisenberg's responded equation of the neutron density in the reflector to O. Hahn's question at Farm Hall[28]. By using the Eq. (B15), a value of γ is similarly derived as follows,

$$\gamma = \frac{1}{BR}\left(\frac{D_r}{D_c} - 1\right) \tag{B16}$$

(c) Since, in the Heisenberg's paper of December 6, 1939, Heisenberg had on the one hand discussed only on the case of $D_r = D_c$, applying this concept to Eq. (B14),

$$\gamma = \frac{\kappa_r}{B} = constant \tag{B17}$$

is given. It should be noted that Eq. (8) of the text can be proven only in the approximations of (a) and (c).

Since all equations of γ: Eqs. (B7), (B14) and (B16), except for the Eq. (B17), are the function of R, it is generally unreasonable that a critical core radius of a next device can be considered to be forecasted from the condition of γ=constant. The fact that Heisenberg had given Eq. (B17) is considered to mean that he had disregarded an important effect of the reflector to a critical mass.

Only the following experimental results are given from the subcritical Haigerloch device,

$$\begin{gathered} \text{core radius: R} = 71 \text{ cm} \\ \text{Z} = 6.7 \\ \text{'Diffusionslänge'} = \frac{1}{\text{B}} : \text{'etwa' 35 cm} \end{gathered} \tag{B18}$$

A calculation of the condition case of γ=constant is first shown. Using the values of R=71 cm, Z=6.7, and 1/B=35 cm to Eq. (B10), γ=0.995 is given. However, since Heisenberg gave γ=0.824, making allowance for experimental uncertainty of 'Diffusionslänge' because of